227 (2023) 211819

Contents lists available at ScienceDirect

Geoenergy Science and Engineering

journal homepage: www.sciencedirect.com/journal/geoenergy-science-and-engineering

Probabilistic forecasting of cumulative production of reservoir fluid with uncertain properties

Lívia Paiva Fulchignoni [a], Christiano Garcia da Silva Santim [b,*], Daniel M. Tartakovsky [a]

[a] Stanford University, Energy Science and Engineering Department, Stanford, CA 94305, USA
[b] ISDB FlowTech, Rio de Janeiro, RJ, Brazil

ARTICLE INFO

Keywords:
Uncertainty quantification
Monte Carlo simulation
Sensitivity analysis
Sobol's indices
Pipe flow

ABSTRACT

Offshore development requires large investments, which have to be made in the presence of multiple sources of uncertainty. Quantification of uncertainty in predictions of a reservoir's production and, consequently, the project's revenue alleviates some of the risks and facilitates more informed business decisions. Despite significant advances in the field of uncertainty quantification, it is still common practice in the industry to rely on most likely parameters for the wellbore and pipeline multiphase flow models when making predictions for the project design. We focus on predictive uncertainty of pipe-flow models, which are used to forecast the cumulative production of an oil reservoir whose fluid properties are typically unknown during the exploration phase. The uncertain inputs of a flow model are treated as random variables with a multivariate Gaussian probability density; the model's prediction of cumulative production is given in term of its distribution, which is estimated via Monte Carlo with Latin hypercube sampling. A global sensitivity analysis is performed to identify the model inputs contributing most to the predictive uncertainty.

1. Introduction

Energy companies make investment decisions based on technical and economic viability studies of exploration fields. The financial evaluation of revenues is mainly dictated by the estimated production, i.e., the oil and gas (O&G) flow rates that a reservoir can provide through its lifetime. This process is informed by simulations of multiphase flow in the reservoir, wellbore and pipelines. For this reason, such simulators are an essential tool during the design phase of a production system. Since many model parameters must be specified to run a multiphase-flow simulator, and since the values of these parameters are inherently uncertain, it is wise to evaluate production flow rates under a probabilistic approach and to quantify the uncertainty associated with their predicted values. This strategy renders the projected revenue a random variable, whose value one can calculate for the required degree of certainty, thus enabling one to make investment decisions based on risk and return.

Uncertainty quantification (UQ) gained popularity in the O&G industry in the 1960s (Stoian, 1965) and has been evolving ever since, although at different rates among different disciplines (drilling, reservoir, production, operation, etc.) (Bickel and Bratvold, 2008). For reservoir simulations, UQ is an established practice (Meisingset, 1999; Yang et al., 2022). In fact, it is standard in the industry to report O&G reserve estimations under three categories (proved, probable, and possible), according to their likelihood (probability). Uncertainty in predictions of the accumulation volume is ideally tracked over time from exploration through discovery, development, and production (Ross, 2001). In contrast, UQ for multiphase-flow models used in flow assurance has not yet been thoroughly investigated (Klinkert, 2018).

Section 1.1 discusses the sources of uncertainty in these flow models and establishes uncertain parameters (reservoir fluid properties) that are considered in this work. Section 1.2 introduces the Monte Carlo simulation (MCS) technique, reviews the literature for applications in multiphase pipe-flow models, and specifies this paper's goals and contributions to the topic. Section 2 presents the UQ methodology applied to model predictions of cumulative production, including a thorough description of the production system used as a case study. Section 3 discusses the results obtained in this study. Section 4 summarizes major conclusions.

1.1. Sources of uncertainty in flow simulations

Uncertainty in predictions of multiphase-flow models, which represent production from a subsurface reservoir to the land-surface facilities, arises from multiple sources. *Structural or model uncertainty* is

* Corresponding author.
E-mail address: chris.santim@isdbflowtech.com (C.G.S. Santim).

https://doi.org/10.1016/j.geoen.2023.211819
Received 24 October 2022; Received in revised form 15 March 2023; Accepted 18 April 2023
Available online 29 April 2023

due to inevitable approximations of "reality" introduced by a model. Various models, such as the drift-flux (Zuber and Findlay, 1965; Wallis, 1969; Ishii, 1977) and two-fluid (Bendiksen et al., 1991) models, encode the laws of conservation of mass, momentum and energy, and represent the complex physics with different degrees of fidelity. These equations require empirical constitutive relations of varying validity and generality. Such relations are usually parameterized via small-scale experiments, whereas the flow models are used on a much larger scale. These experiments are commonly carried out on small acrylic pipes, with working fluids other than O&G.

Compositional and "black-oil" fluid models are a representative example of constitutive relations. They are used to predict the thermophysical behavior of a reservoir fluid by relating the properties of oil, gas, and water phases to pressure and temperature. Compositional fluid models are grounded in thermodynamics, but have a number of fitting parameters whose tuning is subjective and may yield different fluid characterizations.[1] In contrast, black-oil fluid models are empirical but require only a few inputs (i.e., density of each phase at standard conditions, watercut, and reservoir gas–oil ratio) to characterize the fluid mixture across a wide range of pressures and temperatures. (Some black-oil formulations might include additional inputs, e.g., the CO_2 mole fraction of surface gas (Fulchignoni et al., 2022; Glaso, 1980).) The largely subjective choice of the constitutive fluid model yields predictions of the solution gas–oil ratio, oil formation volume factors, gas compressibility factor, oil and gas viscosities, and other derived quantities at each pressure and temperature pair in the flow simulation. This choice can significantly impact the predictions of a flow simulator; for example, different choices of the solution gas–oil ratio and gas-liquid drift models significantly influence the calculated liquid holdup profile along the production line and temperature near the outlet (Santim et al., 2020).

Other contributions to the structural uncertainty include the frequent use of a well/reservoir coupling model as boundary condition (Hadgu et al., 1993). In these cases, the reservoir behavior is described through an inflow performance relationship (IPR) curve that relates the reservoir and well bottom hole pressures to production flow rates. Several coupling models have been proposed (Ahmed, 2018), each of which require different uncertain inputs to characterize the reservoir behavior.

Once the governing equations and accompanying constitutive relations have been selected, they have to be parameterized, i.e., the values of the model parameters have to be specified, usually by fitting the model predictions to experimental data. Fig. 1 collates the parameters (inputs) required by a steady-state black-oil flow model, which we use in our numerical experiments because of its simplicity and low computational cost. Data from the well perforation (drill-bit diameter, deviation profile, casing diameter and thickness, etc.), completion (tubing diameter, thickness, and roughness, gas lift, electric submersible pumps, etc.), flow lines and riser (layout, diameter, thickness, roughness and thermal insulation, etc.) and subsea equipment (separator, pump, etc.) are needed to describe fluid flow and heat transfer. Reservoir data (pressure, temperature, and productivity index) characterizes the source boundary condition. Finally, fluid data – oil, gas, and water densities, gas–oil ratio, and watercut – are inputs to the fluid models. Additional information, such as the surface boundary conditions, is necessary to run a simulation. The need to specify numerical values for all of these inputs introduces *parametric uncertainty* into the modeling process.

Model parameters differ by both their degree of uncertainty and their impact on the model's prediction uncertainty. For instance, frequent measurement of reservoir pressure through pressure buildup tests is expensive; infrequent measurements, the industry standard, increase the uncertainty in this important boundary condition. Conversely, the pipeline diameter is directly measurable and, thus, uncertainty in its value is comparatively low, arising primarily from measurement errors. Within the probabilistic framework, this statement is equivalent to saying that the standard deviation of reservoir pressure, σ_p, exceeds that of pipe diameter, σ_d. Model nonlinearity implies that the condition $\sigma_p \gg \sigma_d$ does not automatically mean that uncertainty in reservoir pressure has larger impact on prediction uncertainty of the pipe-flow simulations than uncertainty in pipe diameter does. Addressing the latter issue falls under the purview of global sensitivity analysis, which is discussed in Section 2.3.

Our study focuses on parametric uncertainty associated with fluid properties, which is prevalent during the exploration phase, when little site-specific information is available. Fluid properties data are collected from a few exploration/appraisal wells. In the absence of such wells, the reservoir fluid is modeled based on information from a basin model whose parameterization relies on data from analogous geological areas. Even when samples are available, their analysis is carried out in laboratory conditions that be quite different from the reservoir conditions (Peña Díez et al., 2022). For these reasons, we treat as uncertain the following fluid-model parameters: oil and gas specific gravities at standard conditions (γ_o and γ_g, respectively), reservoir fluid gas–oil ratio, and watercut. We assume the water density at standard condition to be known with certainty, even though the water properties depend on water salinity, which can be uncertain. Furthermore, density and viscosity of each phase throughout the flow (at varying pressure and temperature conditions) are computed through the black-oil model, whose uncertainties are out of the scope of this study. Our primary goal is to quantify the influence of parametric uncertainty in the fluid model on prediction uncertainty of the cumulative production.

1.2. Uncertainty quantification for flow simulations

While many computationally efficient alternatives to Monte Carlo simulations (MCS) have been used to evaluate prediction uncertainty of models of multiphase flow in heterogeneous porous media (Rotondi et al., 2006; Taverniers et al., 2020; Yang et al., 2022), comparable UQ efforts for multiphase flow in O&G pipes are scarce and mostly limited to MCS. Examples of the latter include quantification of uncertainty in predictions of a two-phase pipe-flow simulator (pressure drop and liquid holdup in an experimental pipe) with uncertain/random inputs (flow rates, viscosities, and densities of both phases; diameter, inclination angle, and length of the pipe; surface tension; and parameters of a non-Newtonian fluid model) (Picchi and Poesio, 2017).

In real production scenarios, MCS were deployed to quantify uncertainty in model predictions of time-varying oil-flow rates from multiple producer wells during the optimization of a field layout design, with the initial flow rates and the reservoir's decline rate acting as uncertain inputs (Sales et al., 2018). They were also used to evaluate the impact of the watercut uncertainty on model predictions of the oil production flow rate (Monteiro et al., 2020), and to analyze the influence of uncertainty in the values of three input parameters (outlet pressure, ambient temperature, and wall roughness) on predictions of the pressure drop and liquid holdup (Klinkert, 2018).

We use an accelerated version of MCS, which relies on Latin hypercube sampling (Taverniers and Tartakovsky, 2020), to investigate the impact of parametric uncertainty in the fluid model on model predictions of the time-dependent production flow rate during the exploration phase, in the context of flow assurance. The analysis is performed for an offshore production scenario, based on a real well located in Campos Basin, Brazil. A result of our analysis is the probabilistic forecast of cumulative oil production and, consequently, the project's revenue.

[1] For example, the choice of characterization of the heaviest hydrocarbon fraction, and splitting and lumping procedures, drastically affect the final fluid model.

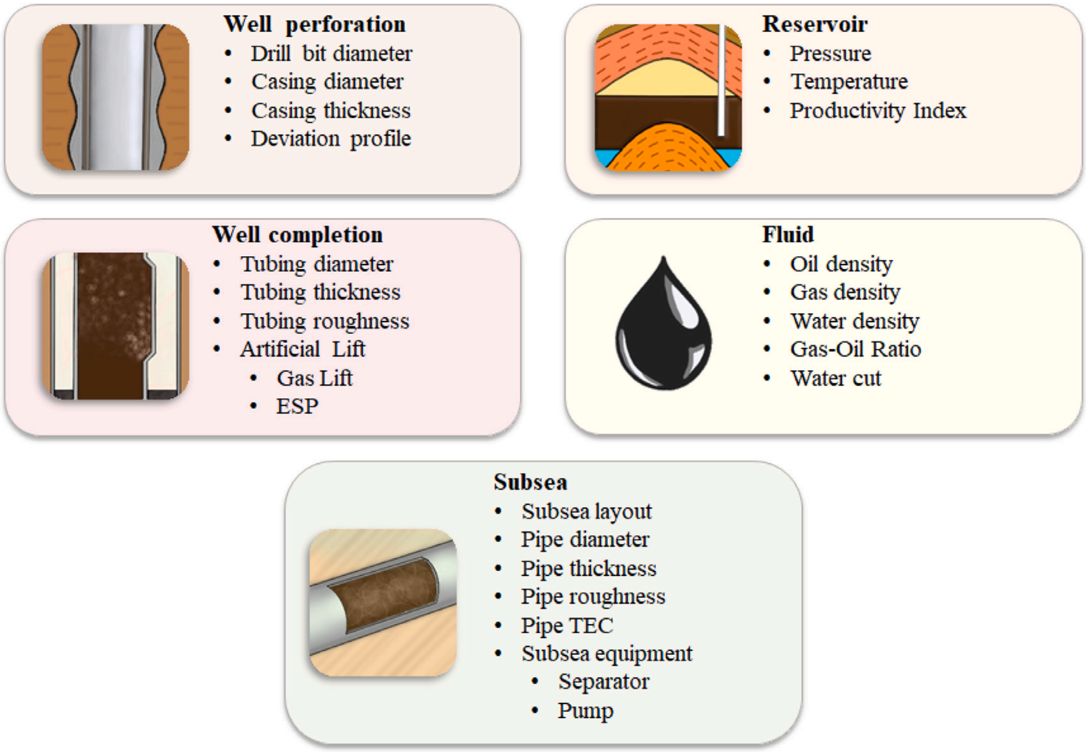

Fig. 1. Physical properties that provide input for a steady-state black-oil flow simulation.

1.3. Global sensitivity analysis

Global sensitivity analysis (GSA) is a distinct and complementary facet of uncertainty management and risk assessment (Ciriello et al., 2017). It seeks to rank the random inputs by their relative contribution to uncertainty in predictions of a quantity of interest (QoI). Depending on whether prediction uncertainty is represented in terms of the QoI's variance or full distribution, GSA can be classified as variance- or distribution-based, respectively; the latter is applicable to a wider range of inputs, while the former is easier to compute (Ciriello et al., 2019). Variance-based GSA ranks the input parameters by their Sobol' indices, whose definition derives from the multivariate analysis of variance (ANOVA) (Winter et al., 2006). These indices can be computed via either MCS (Picchi and Poesio, 2017) or polynomial chaos expansions (Strand et al., 2020).

Undergirding the variance-based GSA is the requirement that random inputs are mutually uncorrelated. This is a questionable assumption in the context of fluid modeling, where many of the model parameters are inter-related. To account for this complication, we use the Rosenblatt transform to map the correlated model inputs onto a set of uncorrelated random variables (Ciriello et al., 2019), which are then used to perform a variance-based GSA. This enables us to identify the most influential parameters to the production flow rate output at different stages of production lifetime.

2. Methodology

2.1. Probabilistic characterization of uncertain fluid properties

We focus on uncertainty in four fluid properties $\boldsymbol{X} = (X_1, \ldots, X_4)^\top \in \mathbb{R}^4$: the oil and gas specific gravities ($X_1 = \gamma_o$ and $X_2 = \gamma_g$), the gas–oil ratio (X_3 = GOR), and watercut (X_4 = WC). Statistical properties of these parameters, e.g., their means and variances, are estimated during the exploration phase, when little information about the reservoir field and its fluid is available. For the sake of generalization and interpretability of the results, and in the absence of evidence to the contrary, we use a multivariate Gaussian probability density function (PDF),

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \frac{1}{2\pi|\boldsymbol{\Sigma}|^{1/2}} \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right], \tag{1}$$

to characterize these random inputs. Here, the vector $\boldsymbol{\mu} \in \mathbb{R}^4$ comprises the means of the four fluid properties, and the positive-definite matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{4\times 4}$ is composed of the covariances between these properties.

The mean values in the vector $\boldsymbol{\mu}$ are typically estimated by experts from their knowledge of similar geological areas. As an example, we use the mean oil specific gravity μ_{γ_o}, the mean gas specific gravity μ_{γ_g}, and the mean gas–oil ratio μ_{GOR} (with units $\mathrm{Sm}^3/\mathrm{Sm}^3$), which are the values measured in a PVT analysis of a reservoir fluid that is deemed representative. Following the treatment of Well 03 in Monteiro et al. (2020), we assume the mean watercut $\mu_{\mathrm{WC}}(t)$ (in percentage) to vary linearly with production time t (in months), increasing at a rate of 0.667%/month. In our experiments, we use the following numbers:

$$\boldsymbol{\mu}(t) = (\mu_{\gamma_o}, \mu_{\gamma_g}, \mu_{\mathrm{GOR}}, \mu_{\mathrm{WC}}(t))^\top = (0.944, 0.610, 163.0, 0.667t)^\top. \tag{2}$$

The covariance matrix $\boldsymbol{\Sigma}$ encapsulates the degree of uncertainty in the values of the model parameters and their correlations. Such uncertainty in the fluid description arises from potential heterogeneity of fluid properties across the field. It can vary greatly from field to field, as it is influenced by factors such as data acquisition, production experience, and geological complexity. In the exploration phase, field experience suggests the standard deviation of oil density and gas–oil ratio to be up to 2% and 20% of their mean values, respectively (Meisingset, 1999). We assume gas density to be as uncertain as the oil density and the standard deviation of watercut to be 10% of its mean value, approximately as in Monteiro et al. (2020). The watercut is independent of the reservoir fluid composition. Our correlation analysis of 220 reservoir fluid samples from Brazilian oil fields yields the correlation matrix

$$\boldsymbol{\Sigma}(t) = \begin{pmatrix} \sigma^2_{X_1} & C_{X_1X_2} & C_{X_1X_3} & C_{X_1X_4} \\ C_{X_2X_1} & \sigma^2_{X_2} & C_{X_2X_3} & C_{X_2X_4} \\ C_{X_3X_1} & C_{X_3X_2} & \sigma^2_{X_3} & C_{X_3X_4} \\ C_{X_4X_1} & C_{X_4X_2} & C_{X_4X_3} & \sigma^2_{X_4}(t) \end{pmatrix}$$
$$= \begin{pmatrix} 3.6 \cdot 10^{-4} & -9.4 \cdot 10^{-5} & -1.8 \cdot 10^{-1} & 0 \\ -9.4 \cdot 10^{-5} & 1.5 \cdot 10^{-4} & 2.7 \cdot 10^{-1} & 0 \\ -1.8 \cdot 10^{-1} & 2.7 \cdot 10^{-1} & 1.1 \cdot 10^{3} & 0 \\ 0 & 0 & 0 & 4.5 \cdot 10^{-3} t^2 \end{pmatrix}. \quad (3)$$

This analysis indicates the strong correlation between the inputs, especially between the gas specific gravity and the gas–oil ratio.

2.2. Monte Carlo simulations

MCS is an ensemble-based computation consisting of three steps. First, N realizations, $\boldsymbol{x}_1, \dots, \boldsymbol{x}_N$, of the random input vector $\boldsymbol{X}$ are drawn from the joint PDF in (1). Second, for each realization of the inputs, $\boldsymbol{x}_n$ ($n = 1, \dots, N$), the flow model is solved to obtain the corresponding realizations of the model output and QoIs. Third, the sample statistics of the QoIs, including their PDFs, are computed.

The root mean squared error (RMSE) of an MC estimate of the QoI's mean decays as $1/\sqrt{N}$ (Owen, 2013). This slow convergence rate is due to the purely random sampling in which most of the samples come from the center of a PDF and relatively few sample from the PDF's tails. To accelerate convergence within a given tolerance, we deploy the Latin hypercube sampling, in which the domain of definition of PDF $f_{\boldsymbol{X}}(\boldsymbol{x})$ is subdivided into equal intervals ("strata") and random realizations $\boldsymbol{x}_n$ are drawn from each interval.

2.3. Global sensitivity analysis

The application of variance-based GSA is limited to uncorrelated random inputs. We use the Rosenblatt transform (Rosenblatt, 1952) to map $\boldsymbol{X}$ onto a random vector $\boldsymbol{U} = (U_1, \dots, U_4)^\top \in \mathbb{R}^4$ uniformly and independently distributed over the unit hypercube $[0,1]^4$. Let $F_{X_i|X_1,\dots,X_{i-1}}(x_i|x_1,\dots,x_{i-1})$ be the conditional cumulative distribution function of X_i given $X_1, \dots, X_{i-1}$. The Rosenblatt transform of $\boldsymbol{X} \in \mathbb{R}^4$ is defined as $T(\boldsymbol{X}) = \boldsymbol{U}$ such that

$$u_1 = F_{X_1}(x_1), \quad (4)$$
$$u_2 = F_{X_2|X_1}(x_2|x_1), \quad (5)$$
$$u_3 = F_{X_3|X_1,X_2}(x_3|x_1,x_2), \quad (6)$$
$$u_4 = F_{X_4|X_1,X_2,X_3}(x_4|x_1,x_2,x_3). \quad (7)$$

Because the Rosenblatt transform is bijective, we define

$$g(\boldsymbol{U}) = h \circ T^{-1}(\boldsymbol{U}) = h(\boldsymbol{X}) = \text{QoI}.$$

In our example, QoI is the simulated oil flow rate at each time step. Thus, we perform the GSA on $g(\boldsymbol{U})$. The first-order, S_{X_i}, and total, T_{X_i}, Sobol's indices are thus defined as

$$S_{X_i} = \frac{\text{Var}[\mathbb{E}[g(\boldsymbol{U})|U_i]]}{\text{Var}[g(\boldsymbol{U})]} = \frac{\text{Var}[\mathbb{E}[\text{QoI}|X_i]]}{\text{Var}[\text{QoI}]} \quad (8)$$
$$T_{X_i} = \frac{\mathbb{E}[\text{Var}[g(\boldsymbol{U})|U_{\sim i}]]}{\text{Var}[g(\boldsymbol{U})]} = \frac{\mathbb{E}[\text{Var}[\text{QoI}|X_{\sim i}]]}{\text{Var}[\text{QoI}]} \quad (9)$$

where $U_{\sim i}$ denotes all parameters but U_i; and $\mathbb{E}[\cdot]$ and $\text{Var}[\cdot]$ are the mean and variance operator, respectively. We rely on the R package `sensobol` (Puy et al., 2022) to compute S_{X_i} and T_{X_i}.

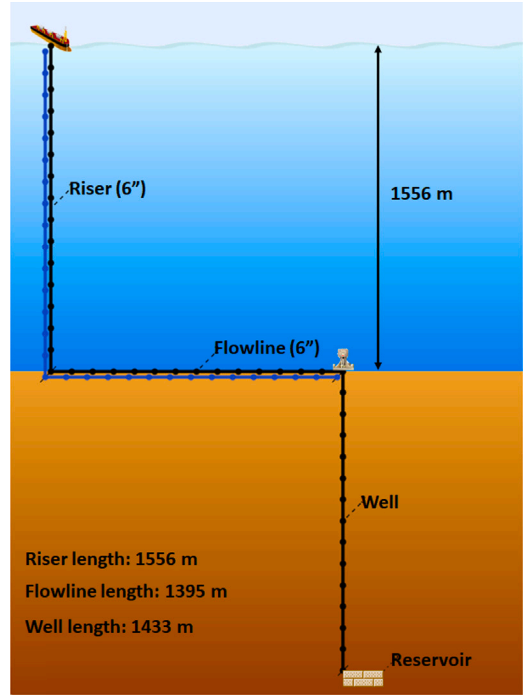

Fig. 2. Schematic of a simplified production well implemented in the flow simulator.

2.4. Case study

We demonstrate our approach on a simplified model of a real offshore production system located in Campos Basin, Brazil, denoted as "Well B" in Oliveira et al. (2017); the well is assumed to produce by natural lift. Fig. 2 shows a schematic representation of the simplified production well in the multiphase flow simulator, along with some general characteristics. The flow simulation boundary conditions are the separator pressure of 16.6 bar; the linear IPR model is used for the reservoir, with the reservoir pressure of 324.1 bar and the productivity index of 53.0 m^3/(d bar). Appendix describes the implemented model, including the fluid and flow constitutive relations, thus providing a benchmark for future studies. This appendix also contains information about the flow behavior at the beginning, middle and end of the lifetime.

Flow simulations are performed with the Petrobras in-house simulator MARLIM II® (Multiphase flow and ARtificial LIft Modeling), which outputs the production flow rates. The production lifetime is simulated in time steps of one month. The reservoir pressure and temperature are assumed to be constant throughout the production lifetime due to the waterflooding process, a secondary recovery technique.

The cumulative oil production is computed via numerical integration over time of the discrete flow rates. For that, we conduct additional MCS, in which flow rates serve as the inputs whose realizations are drawn from the flow-rate PDFs obtained at each time step.

3. Results

3.1. Monte Carlo simulations

Fig. 3 exhibits the results of our convergence study of MCS with Latin hypercube sampling, with the flow rate at the end of the well's lifetime (when the uncertainty on the inputs is higher) playing the role of QoI. The sample mean, μ, and standard deviation, σ, of this QoI converge after $N \approx 5000$ MC realizations (Fig. 3a). This result demonstrates the limited value of the QoI statistics obtained via MCS with $N = 100$ Latin hypercube samples, as done in Monteiro et al.

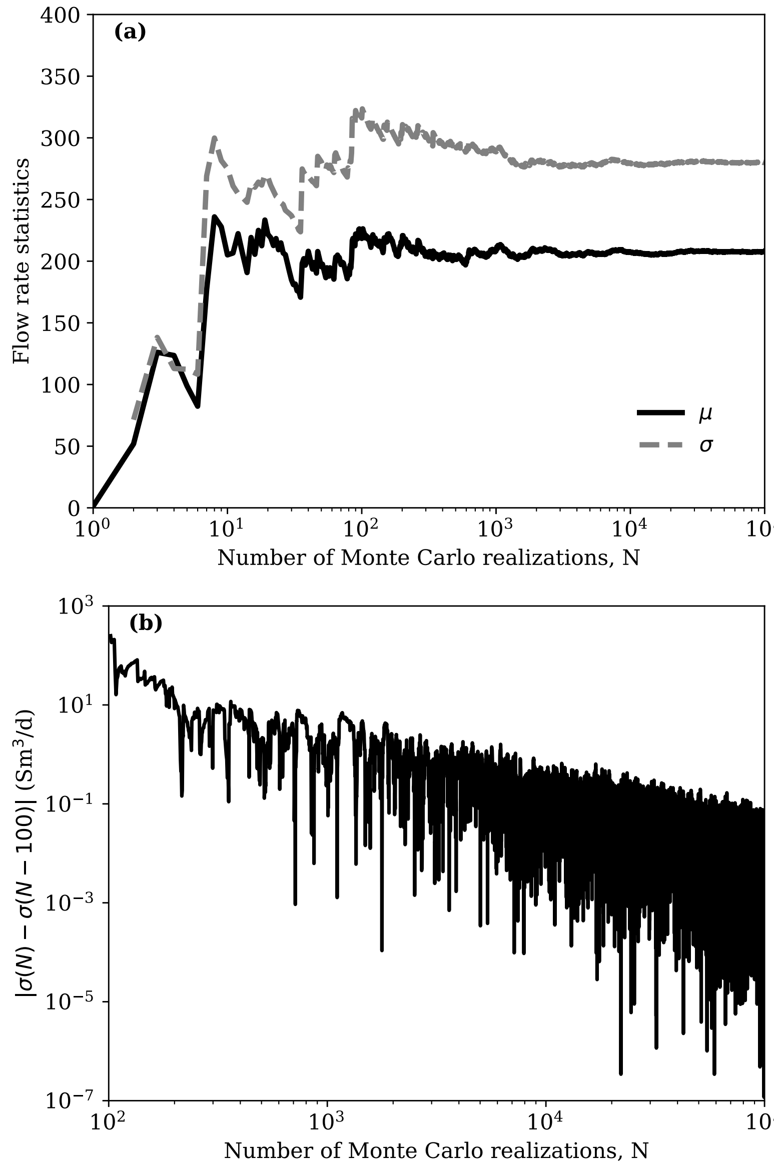

Fig. 3. Convergence analysis of MCS after 10 years of production: (a) sample mean, μ, and standard deviation, σ, of the cumulative production at the end of the well's lifetime as function of the number of MC realizations, N; (b) the absolute difference between two sample standard deviations, $|\sigma(N) - \sigma(N-100)|$, computed from N and $N - 100$ MC realizations.

(2020) for a similar setting. For instance, the mean oil flow rate estimated with $N = 100$ MC realizations is $\mu = 218.4$ Sm^3/d, which is 5% higher than $\mu = 207.7$ Sm^3/d estimated from $N = 10^5$ realizations. This highlights the importance of a convergence analysis of MCS, including the establishment of a convergence criteria that represents the specific goal of a study.

We define such a criteria in terms of the absolute difference between two sample standard deviations, $|\sigma(N) - \sigma(N-100)| \leq \mathcal{E}$, computed from N and $N - 100$ Monte Carlo realizations. For the tolerance level $\mathcal{E} = 1$ Sm^3/d, the MCS convergence is attained after $N \approx 7300$ realizations (Fig. 3b). This is the number of realizations used to obtain the results below. For this N, $\mu = 207.8$ Sm^3/d.

Table 1 reports MC estimates of the descriptive statistics of the oil flow throughout the well's production life. As expected, the mean oil flow rate, μ, decreases as the mean watercut increases throughout the production lifetime, in accordance with Eq. (2). The standard deviation, σ, increases due to higher uncertainty in the watercut, except at Year 10 when more realizations predict zero flow rate. While the PDF of the oil flow rate is symmetric at earlier years, it becomes skewed to the right towards the end of production lifetime (see, also, Fig. 4). The positive skewness of the PDFs at the end of production life reflects the large number of MC realizations predicting no production, due to insufficient reservoir pressure to naturally lift fluid from the formation to surface.

Table 1
Descriptive statistics of the oil flow rate throughout production life.

Production time (years)	Oil flow rate (Sm^3/d)						
	μ	σ	min	P25	P50	P75	max
0	3683.3	115.9	2655.4	3614.9	3691.4	3760.1	3994.4
1	3372.9	108.9	2625.2	3309.6	3380.4	3445.8	3715.0
2	3051.5	118.0	2312.6	2981.2	3059.2	3131.3	3450.4
3	2718.9	140.4	1877.6	2631.7	2725.2	2814.6	3218.9
4	2377.7	169.7	1348.6	2267.3	2382.2	2496.7	3121.2
5	2020.6	203.6	791.4	1883.3	2024.0	2162.4	2729.9
6	1670.4	237.2	0.0	1516.6	1667.3	1826.4	2630.4
7	1302.4	294.5	0.0	1155.0	1326.8	1489.0	2378.6
8	899.6	378.7	0.0	625.6	955.7	1179.2	2095.1
9	488.1	389.1	0.0	171.8	435.4	778.5	1927.1
10	207.8	280.3	0.0	0.0	97.6	333.0	1720.0

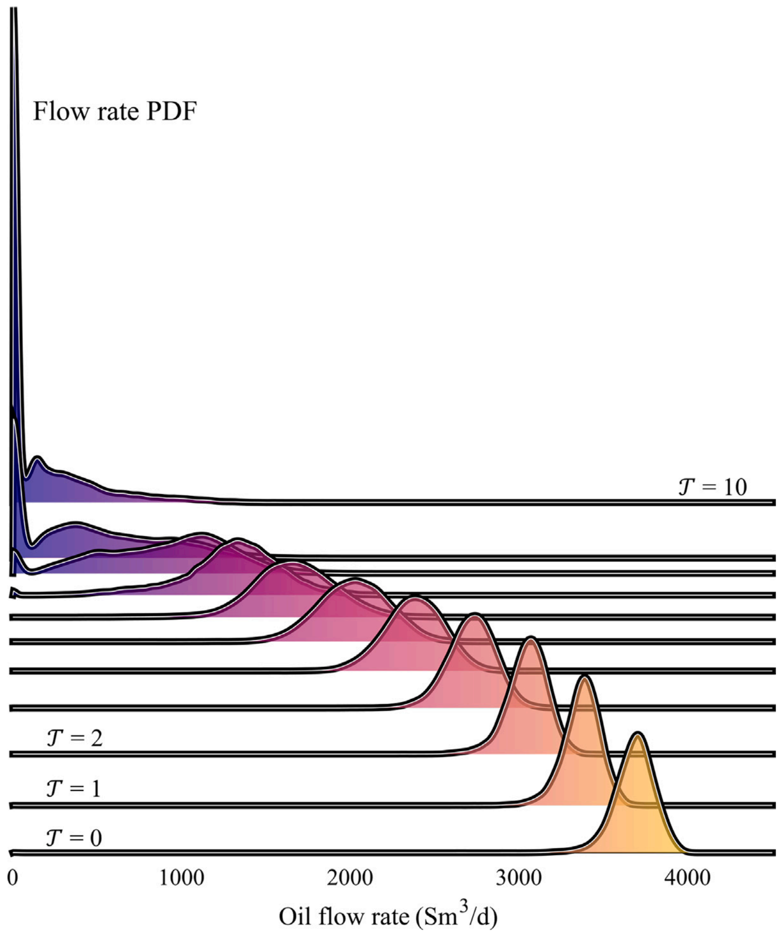

Fig. 4. Probability density function of the oil production flow rate throughout the well's production lifetime, at several times $\mathcal{T}$.

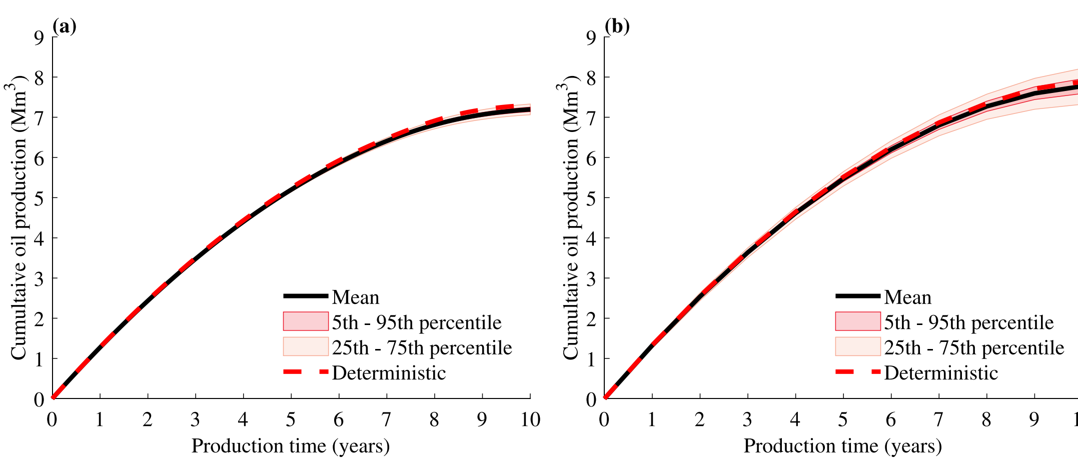

Fig. 5. Cumulative oil production through the well's production lifetime considering as time step for the numerical integration of the flow rates: (a) one month and (b) one year.

It is instructive to compare the probabilistic forecast of the oil flow rate with its deterministic counterpart that uses the expected value of each input in the flow simulation. The latter computation yields the oil flow rates of 3704.4 Sm^3/d at production start (WC = 0), 2057.4 Sm^3/d after 5 years of production (WC = 40%), and 2.0 Sm^3/d after 10 years of production (WC = 80%). (Pressure, temperature, and liquid holdup profiles of these simulations are reported in Appendix.) These deterministic predictions differ significantly from the expected flow rates, especially at the end of production lifetime (Table 1). Furthermore, the deterministic approach lacks any uncertainty quantification associated with its predictions, the information that is just as important as the prediction itself.

Mean cumulative production is computed via numerical integration of the mean flow rates. Its statistics – mean and two confidence intervals – are shown in Fig. 5, together with its deterministic estimate. When the cumulative production is computed from the monthly flow rates (Fig. 5a), its expected value at the end of the well's lifetime is 7.20 Mm^3 and the standard deviation is 0.08 Mm^3, which is 1.1% of the mean. Given that uncertainty in the inputs is significantly higher (20% for gas–oil ratio and 10% for watercut), these results reveal that uncertainty in the values of the reservoir fluid properties is attenuated

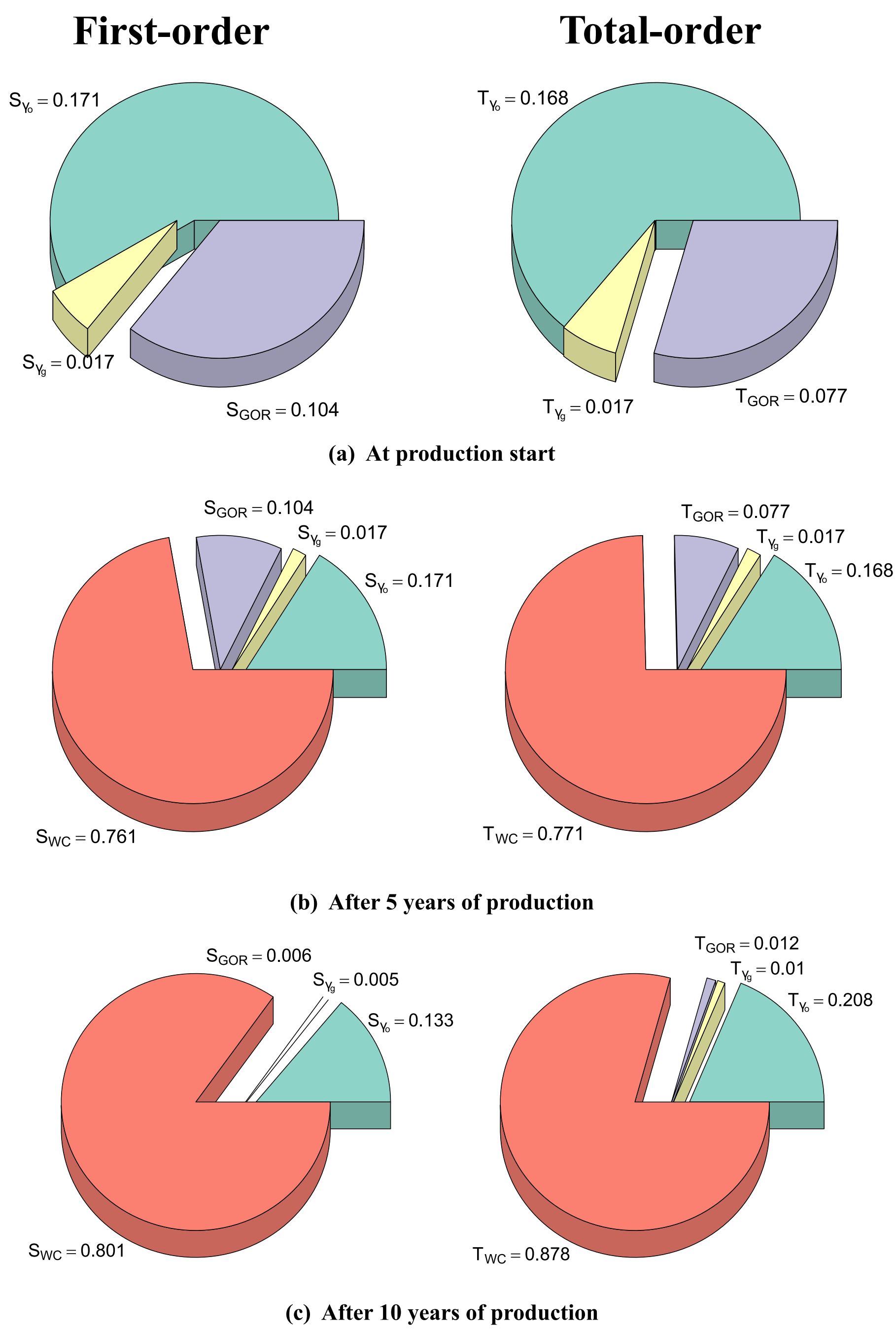

Fig. 6. First-order and total-order Sobol's indices for the input parameters (γ_o, γ_g, GOR, WC) of flow simulation model (a) at the well opening, (b) after five years of production, and (c) after ten years of production.

in the process of computing the cumulative production. The 95th percentile of the total cumulative production is 7.33 Mm^3, which is 0.27 Mm^3 (3.8%) higher than the 5th percentile.

At the well's production lifetime of 10 years, the deterministic computation of the cumulative production (7.30 Mm^3) overpredicts the mean cumulative production (7.20 Mm^3). While this difference might appear to be small, such estimates serve as input to financial evaluations of new production systems projects, in order to quantify the expected revenue and its associated uncertainty. In monetary terms, considering the oil price of the "North Sea Brent" (BRENT) closing value of US$92.36 on 09/01/2022 as the benchmark price, and applying no discount rate to account for the various production times, the deterministic framework overestimates the project's revenues by US$58.6M.

Both expected value and standard deviation of the final cumulative production depend on the time-step size considered for the numerical integration (Fig. 5). Smaller time steps yield smaller standard deviations, i.e., tighter confidence intervals. On the other hand, smaller time steps increase the computational time, which can be prohibitive depending on the application. For example, the use of the yearly flow rates instead of monthly ones reduces the computational cost of the numerical integration of flow rates over 10 years by 92%. It results in the mean cumulative oil production of 7.77 Mm^3 and standard deviation of 0.27 Mm^3 (Fig. 5b). In this case, the deterministic prediction overestimates the project's revenue by US$63.8M. The 5th and 95th percentiles of the total cumulative production are 7.32 Mm^3 and 8.22 Mm^3, respectively. Therefore, the 92% reduction in computational

Table A.1
Parameters and their numerical values used in the multiphase flow simulations.

	Parameter name	Parameter value
Reservoir	Reservoir pressure	324.1 bar
	Reservoir temperature	61.1 °C
	Productivity index	53.0 $m^3/(d\ bar)$
Vertical well	Tubing length	1433 m
	Tubing internal diameter	5.92 in
	Tubing thickness	0.0089 m
	Tubing roughness	0.000178 m
	Casing length	1433 m
	Casing internal diameter	9.66 in
	Casing thickness	0.0138 m
	Cement thickness	0.03 m
	Overall heat transfer coefficient	7.1 $J/(s\ K\ m^2)$
Horizontal flowline	Length	1395 m
	Internal diameter	6 in
	Thickness	0.0783 m
	Overall heat transfer coefficient	5.2 $J/(s.K.m^2)$
Vertical riser	Length	1556 m
	Internal diameter	6 in
	Thickness	0.0488 m
	Overall heat transfer coefficient	8.4 $J/(s\ K\ m^2)$
Ambient and outlet conditions	Soil temperature at reservoir	61.1 °C
	Soil temperature at wellhead	3.9 °C
	Soil temperature gradient	Linear
	Seawater temperature at seabed	3.9 °C
	Seawater temperature at surface	25 °C
	Seawater temperature gradient	Linear
	Seawater current velocity	0.1 m/s
	Pressure at the outlet	16.6 bar
Black-oil fluid	Stock-tank oil specific gravity (γ_o)	0.944
	Stock-tank gas specific gravity (γ_g)	0.610
	Water specific gravity (γ_w)	1.030
	Reservoir fluid gas–oil ratio (GOR)	163.0 Sm^3/Sm^3
	Watercut (WC)	0%

Table A.2
Flow and black-oil constitutive relations used in the multiphase-flow simulations.

		Constitutive relation
Black-oil fluid property	Gas–oil solubility ratio	Standing (1947)
	Oil formation volume factor	Standing (1947)
	Gas compressibility factor	Dranchuk et al. (1973)
	Undersaturated oil viscosity	Vazquez and Beggs (1977)
	Live oil viscosity	Beggs and Robinson (1975)
	Dead oil viscosity	Beggs and Robinson (1975)
	Gas viscosity	Lee et al. (1966)
	Water viscosity	Van Wingen (1950)
	Emulsion relative viscosity	Woelflin tight emulsion (Woelflin, 1942) (inversion watercut of 90%)
Flow		Beggs and Brill (1973) and Palmer (1975)

time corresponds to the 0.19 Mm^3 (US$17.7M) increase in prediction uncertainty (standard deviation) of cumulative production.

The aforementioned difference between the 5th and 95th percentiles of the cumulative production is equivalent to US$24.6M and US$82.8M using the monthly and yearly time steps, respectively. When such large sums of money are involved, which is typical for complex engineering systems, a proper evaluation of the associated risks is essential for making informed business decisions and, ultimately, for the company's success. Hence, it is important that O&G production estimates are performed through the probabilistic approach, where uncertainties are accounted for not only in reservoir flow simulations but also in wellbore and pipeline flow simulations. Attempts to reduce the prediction uncertainty should be considered in view of the cost of information, which may influence arguments in favor or against new data acquisition.

3.2. Sensitivity analysis

Fig. 6 presents Sobol's indices for the flow simulation model at the beginning, middle, and end of production lifetime, i.e., at the well opening, and after five and ten years of production, respectively. The mean watercut at each snapshot is 0%, 40%, and 80%, respectively. The sensitivity of the oil flow rate to the input parameters depends on the production scenario. Here, as production time and, consequently, the expected watercut increase, so does their influence on the predicted oil flow rate. In the early production stages, the oil flow rate is more sensitive to the oil specific gravity, while at the end of the production lifetime it is more sensitive to the watercut. The fact that the well produces an emulsion, whose viscosity significantly increases with watercut (according to Woelflin's (Woelflin, 1942) tight emulsion model), contributes to the great influence of watercut at later production stages. This result suggests that the use of subsea demulsifying strategies may enhance oil production (Oliveira et al., 2017).

Throughout production lifetime, the variable with lowest effect on the predictive uncertainty of the oil flow rate is the gas specific gravity, which means that the multiphase model is less sensitive to variations in this input than to changes in oil density, gas–oil ratio or watercut. Since first-order and total-order indices are similar, the model has no significant interaction effects between γ_o, γ_g, gas–oil ratio, and watercut.

4. Conclusion

Uncertainty quantification plays a key role in decision making for O&G projects, especially in risk assessment of its heavy investments. While uncertainty quantification has been an established practice in reservoir simulation, it has not been thoroughly investigated for multiphase flow models used in flow assurance. This paper quantifies uncertainty in predictions of the production flow rate over the life time of a simplified real production well and, consequently, of its cumulative production. The sources of uncertainty are oil and gas specific gravities, reservoir fluid gas–oil ratio, and watercut. Uncertainty of such inputs is modeled through a multivariate Gaussian probability density function, while uncertainty in the flow simulation output (i.e., the production flow rate) is quantified via Monte Carlo simulations with Latin hypercube sampling. A global sensitivity analysis is performed to identify which of the four fluid parameters affect most the overall predictive uncertainty. Major conclusions are summarized below:

- The sensitivity of the simulated oil flow rate to input parameters depends on the production stage. As production time and, consequently, the expected watercut increase, so does their influence on the flow rate. By the end of the well's lifetime, watercut dominates the flow rate predictive uncertainty, followed by oil density. Conversely, gas density is the input variable with lowest effect on the predictive uncertainty of the oil flow rate throughout the whole production lifetime.
- Convergence properties of Monte Carlo simulations were discussed. The sample size depends on the goal of the study, i.e., on the acceptable error tolerance in predictions of a quantity of interest.
- Flow rate probability distributions change over production time, not only in their parameters but also in shape. While they are symmetric at early production stages, they are skewed to the right at later stages due to the higher number of realizations that result in a zero flow rate. This asymmetry of the output distribution happens despite of the symmetry of the inputs distribution. Also, the expected oil flow rate decreases with production time due to watercut increase.
- Cumulative production estimates depend on the time step used in the numerical integration of computed flow rates. Its expected value and standard deviation at the end of the well's lifetime are 7.20 Mm^3 and 0.08 Mm^3, respectively, considering a monthly time step. For a yearly time step, these statistics are 7.77 Mm^3 and 0.27 Mm^3, respectively. These results indicate that uncertainties associated with fluid properties are attenuated when computing the cumulative production.
- The deterministic framework overestimates production when compared to the probabilistic framework while suggesting certainty. This fact highlights the importance of the probabilistic assessment of estimates from pipe flow simulation models.
- Uncertainty quantification of reservoir fluid properties is particularly important during the design phase of a reservoir development project, when field data of fluid properties or pressure and temperature in the wellbore and pipeline are not available. Once field data is available, they can be used to restrict the uncertainty on the simulation inputs parameters through data assimilation techniques.

The analyses presented here are based on a simplified case study. We did not consider a minimum production rate for the well to be operating, i.e., a minimum revenue that covers operational costs and prevents the well from being shut down, which would be a likely assumption in a real case scenario. The presence of flow instabilities (such as severe slugging) that could reduce production was not evaluated. Changes in fluid composition, i.e., in oil and gas densities and gas–oil ratio, may affect the reservoir's relative permeability (Young, 2022) and consequently the reservoir's productivity index. This and other-second order interactions were also not considered in this work. They are expected to increase the uncertainty in predictions of the quantity of interest even further, and should be a focus of future research.

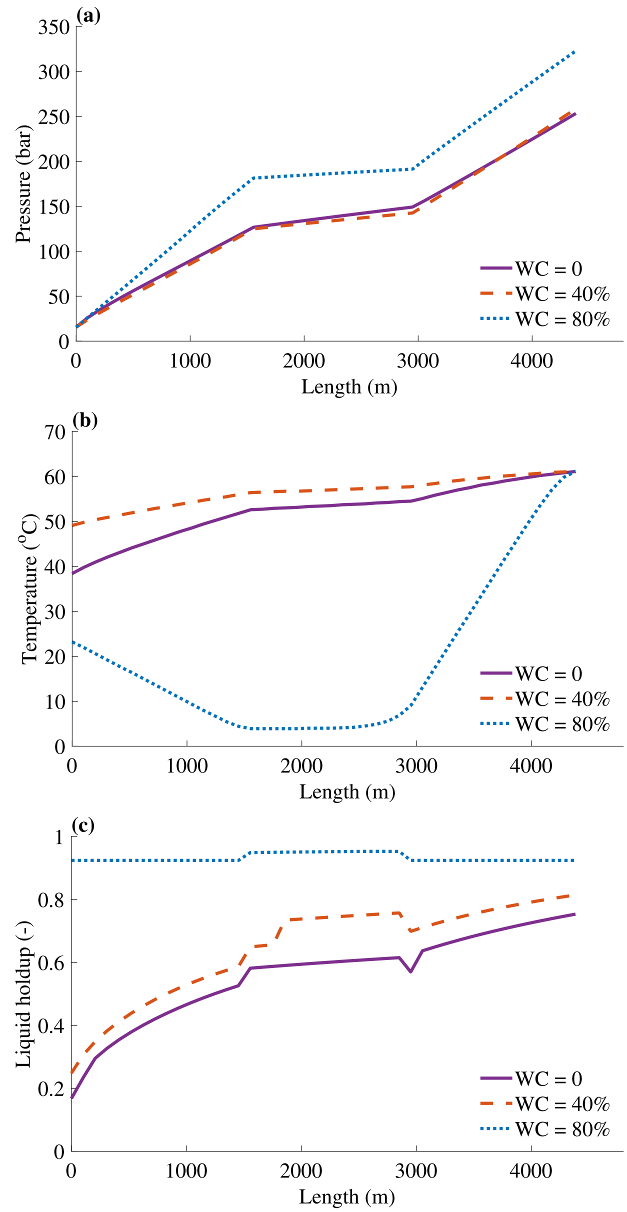

Fig. A.1. (a) Pressure, (b) temperature, and (c) liquid holdup behaviors at the steady-state regime in the wellbore and production line for watercut values of 0%, 40%, and 80%, representing the beginning, middle and end of the production lifetime, respectively.

CRediT authorship contribution statement

Lívia Paiva Fulchignoni: Conceptualization, Methodology, Software, Investigation, Writing – original draft. **Christiano Garcia da Silva Santim:** Investigation, Writing – review & editing. **Daniel M. Tartakovsky:** Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

We would like to thank Petrobras for the data and permission to publish this paper. Some of the computing for this project was performed on the Sherlock cluster at Stanford University.

Appendix. Production flow model

Table A.1 fully describes the simplified production system represented in the multiphase flow simulation implemented in this work. For the fluid parameters treated as random variables, the mean values of their respective probability distributions at the beginning of the production life are reported. Table A.2 shows the flow and fluid correlations considered in the simulation.

Fig. A.1presents the simulated flow behavior at the beginning, middle and end of the production lifetime, considering the mean watercut values of 0%, 40%, and 80%, respectively. The pressure, temperature, and liquid holdup profiles along the wellbore and pipelines are reported.

References

Ahmed, T., 2018. Reservoir Engineering Handbook. Gulf Professional Publishing.

Beggs, D.H., Brill, J.P., 1973. A study of two-phase flow in inclined pipes. J. Pet. Technol. 25 (05), 607–617.

Beggs, H.D., Robinson, J.R., 1975. Estimating the viscosity of crude oil systems. J. Pet. Technol. 27 (09), 1140–1141.

Bendiksen, K.H., Maines, D., Moe, R., Nuland, S., 1991. The dynamic two-fluid model OLGA: Theory and application. SPE Prod. Eng. 6 (02), 171–180.

Bickel, J.E., Bratvold, R.B., 2008. From uncertainty quantification to decision making in the oil and gas industry. Energy Explor. Exploitat. 26 (5), 311–325.

Ciriello, V., Lauriola, I., Bonvicini, S., Cozzani, V., Federico, V.D., Tartakovsky, D.M., 2017. Impact of hydrogeological uncertainty on estimation of environmental risks posed by hydrocarbon transportation networks. Water Resour. Res. 53 (11), 8686–8697. http://dx.doi.org/10.1002/2017WR021368.

Ciriello, V., Lauriola, I., Tartakovsky, D.M., 2019. Distribution-based global sensitivity analysis in hydrology. Water Resour. Res. 55 (11), 8708–8720. http://dx.doi.org/10.1029/2019WR025844.

Dranchuk, P., Purvis, R., Robinson, D., 1973. Computer calculation of natural gas compressibility factors using the standing and katz correlation. PETSOC Ann. Tech. Meet. http://dx.doi.org/10.2118/73-112.

Fulchignoni, L.P., Almeida, A.R., da Fonseca, R., 2022. A novel model for the solution gas-oil ratio suitable for CO_2-rich reservoir fluids. Results Eng. 100681. http://dx.doi.org/10.1016/j.rineng.2022.100681.

Glaso, O., 1980. Generalized pressure-volume-temperature correlations. J. Pet. Technol. 32 (05), 785–795.

Hadgu, T., Zimmerman, R.W., Bodvarsson, G.S., 1993. Coupling of a reservoir simulator and a wellbore simulator for geothermal applications.

Ishii, M., 1977. One-Dimensional Drift-Flux Model and Constitutive Equations for Relative Motion Between Phases in Various Two-Phase Flow Regimes. Technical Report, Argonne National Lab., Ill.(USA).

Klinkert, J., 2018. The characterization of uncertainty for steady state multiphase flow models in pipelines.

Lee, A.L., Gonzalez, M.H., Eakin, B.E., 1966. The viscosity of natural gases. J. Pet. Technol. 18 (08), 997–1000.

Meisingset, K., 1999. Uncertainties in reservoir fluid description for reservoir modeling. SPE Reserv. Eval. Eng. 2 (05), 431–435.

Monteiro, D.D., Duque, M.M., Chaves, G.S., Ferreira Filho, V.M., Baioco, J.S., 2020. Using data analytics to quantify the impact of production test uncertainty on oil flow rate forecast. Oil Gas Sci. Technol.–Revue D'IFP Energies Nouvelles 75, 7.

Oliveira, M.C.K., Paiva, L.F., Meireles, F.A., Mendes, R., Silva, P.M.D., Machado, L.G., 2017. Subsea demulsifier injection to reduce emulsion viscosity and enhance crude oil production. In: Offshore Technology Conference.

Owen, A.B., 2013. Monte Carlo Theory, Methods and Examples.

Palmer, C.M., 1975. Evaluation of Inclined Pipe Two-Phase Liquid Holdup Correlations Using Experimental Data (Master's thesis). University of Tulsa.

Peña Díez, J.L., Merino García, D., Benito Iglesias, V., Sanz Sanz, P., 2022. Computer implemented method for characterizing a target fluid of a hydrocarbon reservoir under uncertainty. US Patent 11, 519, 897.

Picchi, D., Poesio, P., 2017. Uncertainty quantification and global sensitivity analysis of mechanistic one-dimensional models and flow pattern transition boundaries predictions for two-phase pipe flows. Int. J. Multiph. Flow. 90, 64–78.

Puy, A., Piano, S.L., Saltelli, A., Levin, S.A., 2022. sensobol: an R package to compute variance-based sensitivity indices. J. Stat. Softw. 102 (5), 1–37. http://dx.doi.org/10.18637/jss.v102.i05.

Rosenblatt, M., 1952. Remarks on a multivariate transformation. Ann. Math. Stat. 23 (3), 470–472.

Ross, J.G., 2001. Guidelines for the Evaluation of Petroleum Reserves and Resources. Society of Petroleum Engineers Richardson, USA.

Rotondi, M., Nicotra, G., Godi, A., Contento, F.M., Blunt, M.J., Christie, M., et al., 2006. Hydrocarbon production forecast and uncertainty quantification: A field application. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers.

Sales, L.d.P.A., Pitombeira-Neto, A.R., de Athayde Prata, B., 2018. A genetic algorithm integrated with Monte Carlo simulation for the field layout design problem. Oil Gas Sci. Technol.–Revue D'IFP Energies Nouvelles 73, 24.

Santim, C.G.S., Fulchignoni, L.P., Rosa, E.S., Gaspari, E.F., 2020. Transient multiphase flow modeling and validation in a real production system with high CO_2 content using the drift-flux model. J. Pet. Sci. Eng. 188, http://dx.doi.org/10.1016/j.petrol.2020.106903.

Standing, M., 1947. A pressure-volume-temperature correlation for mixtures of California oils and gases. In: Drilling and Production Practice. OnePetro.

Stoian, E., 1965. Fundamentals and applications of the Monte Carlo method. J. Can. Pet. Technol. 4 (03), 120–129.

Strand, A., Smith, I.E., Unander, T.E., Steinsland, I., Hellevik, L.R., 2020. Uncertainty propagation through a point model for steady-state two-phase pipe flow. Algorithms 13 (3), 53.

Taverniers, S., Bosma, S.B.M., Tartakovsky, D.M., 2020. Accelerated multilevel Monte Carlo with kernel-based smoothing and Latinized stratification. Water Resour. Res. 56 (9), http://dx.doi.org/10.1029/2019WR026984, e2019WR026984.

Taverniers, S., Tartakovsky, D.M., 2020. Estimation of distributions via multilevel Monte Carlo with stratified sampling. J. Comput. Phys. 419, 109572. http://dx.doi.org/10.1016/j.jcp.2020.109572.

Van Wingen, N., 1950. Viscosity of oil, water, natural gas, and crude oil at varying pressures and temperatures. The Secondary Recovery of Oil in the United States.

Vazquez, M., Beggs, H.D., 1977. Correlations for fluid physical property prediction. In: SPE Annual Technical Conference and Exhibition. http://dx.doi.org/10.2118/6719-MS.

Wallis, G., 1969. One-Dimensional Two-Phase Flow. McGraw-Hill, New York, USA.

Winter, C.L., Guadagnini, A., Nychka, D., Tartakovsky, D.M., 2006. Multivariate sensitivity analysis of saturated flow through simulated highly heterogeneous groundwater aquifers. J. Comput. Phys. 217 (1), 166–175.

Woelflin, W., 1942. The viscosity of crude-oil emulsions. In: Drilling and Production Practice. OnePetro.

Yang, H.J., Tchelepi, H.A., Tartakovsky, D.M., 2022. Method of distributions for two-phase flow in heterogeneous porous media. Water Resour. Res. 58 (12), http://dx.doi.org/10.1029/2022WR032607, e2022WR032607.

Young, L.C., 2022. Compositional reservoir simulation: a review. SPE J. 1–47.

Zuber, N., Findlay, J.A., 1965. Average volumetric concentration in two-phase flow systems. J. Heat Transf.-Trans. ASME 87, 453–468.